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Black string's tunnelling radiation

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Abstract

We use the Parikh–Wilczek method to study the tunnelling radiation from the event of the black string, which is asymptotically anti-de Sitter and possesses cylindrical symmetry. We show that higher corrections to the semi-classical rate which are caused by energy conservation and angular momentum conservation exist and the emission rate has completely the same functional form as that for spherically symmetric or axisymmetric black holes.

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Hawking [1] proved that the black hole could radiate particles quantum-mechanically. Since Gibbons and Hawking [2] demonstrated that the radiation is exactly thermal, much work to prove that the energy spectrum is precisely thermal spectrum has been done [3–6]. However, there are two puzzles: one is where the barrier appears during the radiation. The other is the purely thermal spectrum, from which we cannot obtain any other information but one parameter, i.e. temperature, this means that if things are absorbed into the black hole, then their important information such as unitarity will be lost during the emission and there will be no marks left once the black hole is evaporated out. Recently, Parikh and Wilczek [7–9] proposed a method to calculate the emission rate at which particles tunnel across the event horizon. They treat Hawking radiation as a tunnelling process. They find that the barrier is created by the outgoing particle itself, and their key insight is to find a coordinate system which is well behaved at the event horizon to calculate the emission rate. In this way, they have calculated the corrected emission spectrum of the spherically symmetric black holes, such as Schwarzschild black holes and Reissner–Norström black holes. Last year, this method was used to calculate the emission rate of particles from other spherically symmetric black holes [10–14] and also extended to investigate the tunnelling radiation from axisymmetric black holes [15–18]. In this paper, we wish to extend this method to a black string and calculate the corrected emission spectrum of particles from its event horizon. This background is chosen for several reasons: it possesses cylindrical symmetry, it represents a system which may physically exist in the universe and there has also been a recently revived interest in anti-de Sitter spacetime in the context of conformal field theories.

In Boyer–Lindquist coordinates, the line element in a black cosmic string spacetime can take the following form: [19]

$$\mathrm{d}s^2 = -\frac{1}{\alpha^2\rho^2}\left[\Delta_\rho - \frac{2(1-\lambda)}{1+\lambda}\Sigma_\rho\right]\mathrm{d}t^2 + \frac{\alpha^2\rho^2}{\Delta_\rho}\,\mathrm{d}\rho^2 + \alpha^2\rho^2\,\mathrm{d}z^2 + \frac{1}{\alpha^4\rho^2}(\Delta_\rho + \Sigma_\rho)\,\mathrm{d}\varphi^2 - \frac{8a\Sigma_\rho}{3\alpha^2\rho^2(1+\lambda)}\,\mathrm{d}t\,\mathrm{d}\varphi, \tag{1}$$

where

$$\Sigma_\rho = 2(1+\lambda)M\alpha\rho - 4Q^2, \qquad \alpha^2 = -\frac{1}{3}\Lambda\ (\alpha > 0),$$
$$\Delta_\rho = \alpha^4\rho^4 - 2(3\lambda-1)M\alpha\rho + \frac{4(3\lambda-1)}{\lambda+1}Q^2, \tag{2}$$
$$\lambda = \sqrt{1 - \frac{8a^2\alpha^2}{9}}, \qquad a = \frac{J}{M}.$$

Here M, Q and J are the Arnowitt–Deser–Misner (ADM) mass, charge and angular momentum per unit length of the black string in z direction, respectively, Λ is the cosmological constant.

The horizons are determined by

$$\Delta_\rho = \alpha^4\rho^4 - 2(3\lambda-1)M\alpha\rho + \frac{4(3\lambda-1)}{\lambda+1}Q^2 = 0. \tag{3}$$

When $\lambda > 1/3$ and $Q^2 \leqslant (3/8)(\lambda+1)M^{4/3}(3\lambda-1)^{1/3}/2^{1/3}$, equation (3) has two positive real roots. We use ρ_H to denote the position of the outer horizon.

The Bekenstein–Hawking entropy (BHE) per unit length of the black string in z direction is

$$S_{\mathrm{BH}} = \frac{A_H}{4} = \frac{\pi\alpha\rho_H^2}{2}\sqrt{\frac{\lambda+1}{3\lambda-1}}, \tag{4}$$

where $A_H = \iint\{\sqrt{g_{22}\,g_{33}}\}_{\rho=\rho_H}\,\mathrm{d}z\,\mathrm{d}\varphi$ is the horizon area.

In the dragging coordinates, the spacetime line element is

$$\mathrm{d}s^2 = -\tilde{g}_{00}\,\mathrm{d}t^2 + \frac{\alpha^2\rho^2}{\Delta_\rho}\,\mathrm{d}\rho^2 + \alpha^2\rho^2\,\mathrm{d}z^2, \tag{5}$$

where

$$\tilde{g}_{00} = g_{00} - \frac{g_{03}^2}{g_{33}} = -\frac{\alpha^2\rho^2\Delta_\rho}{\Delta_\rho + \Sigma_\rho}. \tag{6}$$

The dragging angular velocity is

$$\Omega_H = -\left.\frac{g_{03}}{g_{33}}\right|_{\rho=\rho_H} = \frac{4a\alpha^2}{3(1+\lambda)}. \tag{7}$$

Equation (7) shows that the dragging angular velocity of a black string does not depend on its mass and horizon radius.

Following the Parikh–Wilczek method, we make the following transformation to obtain a coordinate system, which behaves well at the horizon and which is flat Euclidean space in radial to constant-time slices:

$$\mathrm{d}t = \mathrm{d}T + f(\rho, z)\,\mathrm{d}\rho + g(\rho, z)\,\mathrm{d}z, \tag{8}$$

with the integrability condition

$$\frac{\partial f(\rho, z)}{\partial z} = \frac{\partial g(\rho, z)}{\partial \rho}, \tag{9}$$

and let

$$\frac{\alpha^2\rho^2}{\Delta_\rho} + f^2\tilde{g}_{00} = 1. \tag{10}$$

Then, we obtain a new line element (namely Painlevé one)

$$\begin{aligned} \mathrm{d}s^2 = \tilde{g}_{00}\,\mathrm{d}T^2 + 2\sqrt{\tilde{g}_{00}\left(1-\frac{\alpha^2\rho^2}{\Delta_\rho}\right)}\,\mathrm{d}T\,\mathrm{d}\rho + \mathrm{d}\rho^2 + (\alpha^2\rho^2 + g^2\tilde{g}_{00})\,\mathrm{d}z^2 \\ + 2\tilde{g}_{00}g\,\mathrm{d}T\,\mathrm{d}z + 2g\sqrt{\tilde{g}_{00}\left(1-\frac{\alpha^2\rho^2}{\Delta_\rho}\right)}\,\mathrm{d}\rho\,\mathrm{d}z. \end{aligned} \tag{11}$$

The new coordinate system has a number of attractive features. First, it is well behaved at the horizon. Second, constant-time slices are just flat Euclidean space in radial. Third, ∂_T is a Killing vector in the global spacetime. Finally, the metric in this new coordinates satisfies Landau's condition of coordinate clock synchronization which is given by [20]

$$\frac{\partial}{\partial x^j}\left(-\frac{g_{0i}}{g_{00}}\right) = \frac{\partial}{\partial x^i}\left(-\frac{g_{0j}}{g_{00}}\right), \qquad (i, j = 1, 2, 3). \tag{12}$$

That is, the coordinate clock synchronization in the Painlevé coordinates can be transmitted from one place to another though the line element is not diagonal. In quantum mechanics, it is an instantaneous process that particle tunnels across a barrier. Thus, this feature is necessary for us to discuss the tunnelling process.

The radial outgoing null geodesic is given by

$$\dot{\rho} = \frac{\mathrm{d}\rho}{\mathrm{d}T} = \frac{\alpha\rho\Delta_\rho}{\left(\alpha\rho + \sqrt{\alpha^2\rho^2 - \Delta_\rho}\right)\sqrt{\Delta_\rho + \Sigma_\rho}}. \tag{13}$$

If the particle's self-gravitation, energy conservation and angular momentum conservation are taken into account, when a particle of energy ω and angular momentum $a\omega$ is emitted, the black string's ADM mass and angular momentum will become $M - \omega$ and $a(M - \omega)$, and all of the equations which are related with $r_H(M)$ should be used with $M \to M - \omega$. Since the metric is of cylindrical symmetry, we regard the outgoing particle as a cylindrical shell of energy and angular momentum during the tunnelling process. On the other hand, the coordinate φ does not exist in equation (11), that is, φ is an ignorable coordinate in the Lagrange function. In order to eliminate the freedom of φ, the action of the outgoing particle which crosses the horizon outwards from the initial radius ρ_i to the final radius ρ_j can be written as [15]

$$\begin{aligned} Z &= \int_{t_i}^{t_j} (L - P_\varphi\dot{\varphi})\,\mathrm{d}t = \int_{\rho_i}^{\rho_j} P_r\,\mathrm{d}\rho - \int_{\varphi_i}^{\varphi_j} P_\varphi\,\mathrm{d}\varphi \\ &= \int_{\rho_i}^{\rho_j}\int_0^{P_\rho} \mathrm{d}P_\rho\,\mathrm{d}\rho - \int_{\varphi_i}^{\varphi_j}\int_0^{P_\varphi} \mathrm{d}P_\varphi\,\mathrm{d}\varphi, \end{aligned} \tag{14}$$

where P_ρ and P_φ are the canonical momentum conjugates to ρ and φ. Using the Hamilton's equation

$$\frac{\mathrm{d}\rho}{\mathrm{d}t} = \left.\frac{\mathrm{d}H}{\mathrm{d}P_\rho}\right|_{(\rho,\varphi,P_\varphi)}, \qquad \frac{\mathrm{d}\varphi}{\mathrm{d}t} = \left.\frac{\mathrm{d}H}{\mathrm{d}P_\varphi}\right|_{(\rho,\varphi,P_\rho)}, \tag{15}$$

where

$$\mathrm{d}H|_{(\rho,\varphi,P_\varphi)} = \mathrm{d}M, \qquad \mathrm{d}H|_{(\rho,\varphi,P_\rho)} = \Omega_H\,\mathrm{d}J, \tag{16}$$

we find that equation (14) becomes

$$Z = \frac{3\lambda - 1}{2} \int_{\rho_i}^{\rho_j} \int_{M}^{M-\omega} \frac{\left(\alpha\rho + \sqrt{\alpha^2\rho^2 - \Delta_\rho}\right)\sqrt{\Delta_\rho + \Sigma_\rho}}{\alpha\rho\Delta_\rho} \, \mathrm{d}M \, \mathrm{d}\rho. \tag{17}$$

It is easy to find that $\rho = \rho_H$ is a pole. The integral can be evaluated by deforming the contour around the pole, so as to ensure that positive energy solution decay in time. Note that all real parts, divergent or not, can be discarded since they only contribute a phase. Integrating over ρ, we obtain

$$Z = -\mathrm{i}\sqrt{(3\lambda - 1)(1 + \lambda)}\pi\alpha^2 \int_{M}^{M-\omega} \frac{\rho_H^2}{\Delta'_\rho(r_H)} \, \mathrm{d}M. \tag{18}$$

From the horizon equation $\Delta_\rho(\rho_H) = 0$, we can easily obtain

$$\mathrm{d}M = \frac{\Delta'(\rho_H)}{2(3\lambda - 1)\alpha\rho_H} \, \mathrm{d}\rho_H, \tag{19}$$

upon which the action (18) becomes

$$Z = -\frac{\mathrm{i}\pi\alpha}{2}\sqrt{\frac{1+\lambda}{3\lambda - 1}} \int_{\rho_i}^{\rho_j} \rho_H \, \mathrm{d}\rho_H = -\frac{\mathrm{i}\pi\alpha}{4}\sqrt{\frac{1+\lambda}{3\lambda - 1}}\left(\rho_j^2 - \rho_i^2\right). \tag{20}$$

Adopting the Wentzel–Kramers–Brillouin approximation, the relationship between the tunnelling probability of the particle and the imaginary part of the action is described by $\Gamma \sim \exp(-2 \operatorname{Im} Z)$ [21]. Therefore,

$$\Gamma \sim \exp\left(\frac{\pi\alpha}{2}\sqrt{\frac{1+\lambda}{3\lambda - 1}}\left(\rho_j^2 - \rho_i^2\right)\right) = \exp(\Delta S_{\mathrm{BH}}), \tag{21}$$

where $\Delta S_{\mathrm{BH}} = S_{\mathrm{BH}}(M - \omega) - S_{\mathrm{BH}}(M)$ is the change in the BHE per unit length black string before and after the emission. This result is obviously consistent with an underlying unitary theory.

When $Q = a = 0$, that is, for a neutral and static black string, after expanding ΔS_{BH} in ω we have

$$\begin{aligned} \Gamma \sim \exp(\Delta S_{\mathrm{BH}}) &= \exp\left(-\frac{4^{2/3}\pi}{3\alpha M^{1/3}}\omega - \frac{4^{2/3}\pi}{9\alpha M^{4/3}}\omega^2 + O(\omega^2)\right) \\ &= \exp(-\beta\omega - \beta\omega^2/6M + O(\omega^2)), \end{aligned} \tag{22}$$

where $\beta = (4\pi/3\alpha)(4M)^{-1/3}$ is the inverse temperature. Obviously, the corrected spectrum is not exactly thermal. The leading-order term gives the well-known thermal Boltzmann factor $\mathrm{e}^{-\beta\omega}$. The corrections are indicative of a 'grey-body' factor in the emission spectrum, that is, a deviation from pure thermality [16].

We should note that the emission rate (21) for cylindrically symmetric black string has completely the same functional form as that for spherically symmetric or axisymmetric black holes.

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